Articulatory Parameters in a Dispersion-Focalization Model of Vowel Systems*

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1 Overview

Since the seminal work of Liljencrants and Lindblom (1972), a key testing ground for functional, evolutionary, or emergentist approaches to sound systems has been the typology of vowel inventories (for example, Lindblom 1986, Schwartz et al. 1997a, de Boer 2000).

An important innovation of Schwartz et al.'s Dispersion-Focalization Theory (DFT) was calculating the optimality ("energy") of a vowel system as a weighted combination of *two* separate auditory parameters:

(1) **dispersion**: maximization of the auditory distance between vowels (as in Liljencrants and Lindblom 1972)

focalization: maximization of the importance of "focal" vowels such as [i] and [y].

DFT makes reasonably good predictions, matching or approximating many of the attested vowel systems found in the UCLA Phonological Segment Inventory Database (UPSID; Maddieson 1984, Maddieson and Precoda 1989).

However, there are still numerous attested vowel systems that DFT does not predict to be optimal, most notably, many systems containing [ə] and other more centrally located vowels, which happen to be less articulatorily extreme than more peripheral or focal vowels.

We argue that an articulatory parameter should be added to DFT, and we report promising preliminary results from modifications to DFT which model articulatory effort.

2 How Dispersion-Focalization Theory works

In DFT, vowel systems are compared according to their total "energy" according to the distribution and types of vowels in each system. The lower a vowel system's total energy is, the more optimal it is.

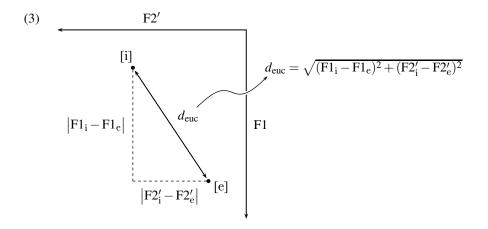
For a given vowel system $\{V_1, ..., V_N\}$, each vowel V_i is characterized by its first four formants $\langle F1_i, F2_i, F3_i, F4_i \rangle$, measured in Bark.¹

A system's total energy E_{DF} (2) is just the simple sum of its dispersion energy E_D (§2.1) and its focalization energy E_F (§2.2):

(2)
$$E_{DF} = E_D + E_F$$

2.1 Dispersion

The dispersion energy of a vowel system is a measure of the overall auditory distance between the vowels in the system. The basic auditory distance between two vowels V_i and V_j is the euclidean distance d_{euc} between them in the auditory space based on their values of F1 and *effective* F2, a.k.a. "F2 prime", a hypothesized perceptual integration of F2, F3, and F4, symbolized as F2' (Carlson et al. 1970, 1975):

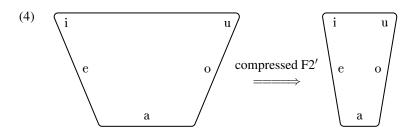


We follow Schwartz et al. in calculating Bark with the formula $f_{\rm Bk} = 7 \cdot \sinh^{-1}(f_{\rm Hz}/650)$.

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Problem: Because F2' spans a significantly larger range (about 10–11 Bk) than F1 does (only about 4–5 Bk), this simple euclidean measure of auditory distance overgenerates color (F2') contrasts in comparison to height (F1) contrasts.

In order to generate more realistic predictions about color vs. height contrasts, phonetic models of vowel dispersion must compress the color space:



There is independent acoustic and perceptual support for weighting F1 more heavily than F2'. For example, F1 is known to be louder than higher formants, and louder formants will weight more heavily in perceptibility (see Lindblom 1986, Schwartz et al. 1997a, Benkí 2003).

The amount of weighting F2' receives is represented in DFT by the parameter λ , which falls between 0 (for which dispersion is determined solely by F1) and 1 (for which F1 and F2' contribute equally to dispersion).

The total dispersion energy E_D of a vowel system with N vowels is the sum of the inverse squares of the λ -weighted distances d_{ij} between every pair of vowels V_i and V_i in the system:

In DFT, the total dispersion energy E_D (5) of a vowel system with N vowels is the sum of the inverse squares of the λ -weighted distances d_{ij} between every pair of vowels V_i and V_j in the system:

(5)
$$E_D = \sum_{\substack{i=1,\dots,N-1\\j=i+1,\dots,N}} \frac{1}{d_{ij}^2} \quad \text{where } d_{ij} = \sqrt{(F1_i - F1_j)^2 + \lambda^2 (F2_i' - F2_j')^2}$$

lower $E_D \Leftrightarrow$ more perceptually peripheral vowel system

2.2 Focalization

DFT additionally assumes that some vowels, so-called "focal vowels", are preferred in vowel systems due to their own inherent acoustic qualities, irrespective of the relational role they play in the system as a whole.

Specifically, a focal vowel in DFT has one or more pairs of adjacent formants that are close together, causing the formants to enhance each other, and making the vowel more perceptually robust overall (Schwartz and Escudier 1987, 1989; cf. Stevens 1972).

The focalization energy E_F of a vowel system is the sum of the focalization energies for each vowel in the system.

Each individual vowel's focalization energy is the negative sum of the inverse squares of the differences between adjacent formants:

(6)
$$E_F = \alpha \sum_{i=1}^{N} \left(\frac{-1}{(F1_i - F2_i)^2} + \frac{-1}{(F2_i - F3_i)^2} + \frac{-1}{(F3_i - F4_i)^2} \right)$$

$$[\text{lower } E_F \Leftrightarrow \text{more focal vowels}]$$

The most focal vowels in DFT by far are [i] and [y], with others ranked roughly as in (7):

(7) low E_F high E_F (high magnitude negative) (low magnitude negative) [i y] < [i] < [e] < [Y] < [E] < [α a α] < [U α α α o α α] < [i α Y] most focal least focal

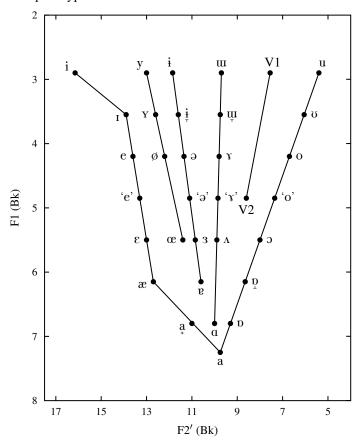
2.3 Prototypes

To limit the amount of computation time required to find optimal vowel systems, Schwartz et al. utilize a finite, predetermined set of 37 vowel "prototypes" (8).

These prototypes are based primarily upon the vowel system from UPSID, with two extra vowels [V1 V2] in the gap in the acoustic vowel space between the back round vowels and the back unrounded vowels. Though Schwartz et al. do not define these vowels physically, we take them to be back vowels with neutral lip positions (neither round nor unround, something like IPA $[\mu, \phi]$ or $[\mu, \phi]$).

For each prototype vowel, Schwartz et al. set fixed values for F1–F4 that are typical of an adult male speaker, with F2' calculated from F2, F3, and F4 by Mantakas et al.'s (1986) computation.

(8) DFT prototypes



2.4 Search algorithm

Searching the total set of possible systems to find the one single optimal system is not a computationally trivial task, even with the limitation of only having 37 vowel prototypes to choose from.

Thus, some search algorithm must be used which picks out only certain vowel systems for consideration of being the most optimal. We use the search algorithm in (9), an improvement over Schwartz et al.'s original search algorithm (see Sanders and Padgett 2008 for discussion):

(9) a. For each value of N = 3, ..., 9, initialize a catalog K_N of all vowel systems of size N already shown to be optimal by Schwartz et al. anywhere in the $\lambda \times \alpha$ space. For example:

$$K_5 = \left\{ \begin{array}{l} \text{[i e a o u], [i y a 'o' u],} \\ \text{[i 'e' a 'o' u], [i ɛ a u u]} \end{array} \right\}$$

This initialized catalog only needs to be created once.

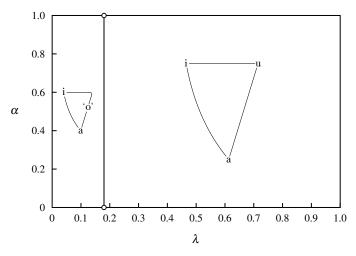
- b. For each value of N, randomly sample 5000 $\langle \lambda, \alpha \rangle$ pairs drawn from $[0,1] \times [0,1]$.²
- c. For each $\langle \lambda, \alpha \rangle$ pair, randomly sample 4,603 candidate vowel systems of size N drawn from the 37 vowel prototypes (this is enough to have a 99% chance of finding a system in the top 0.1% of all possible systems in terms of optimality (lowest energy)). Add to this set of candidates all of the known optimal systems from K_N .
- d. For each $\langle \lambda, \alpha \rangle$ pair and its set of candidate vowel systems, compute the energy of every candidate system, including those from K_N , according to equations (2,5,6).
- e. For each $\langle \lambda, \alpha \rangle$ pair and its set of candidate vowel systems, select as optimal the candidate system with the lowest energy. If this optimal system is not yet in K_N , add it. Otherwise, make no change to K_N .
- f. Repeat steps (b)–(e) five times, and then continue repeating them until K_N no longer changes.

2.5 Visualization of DFT's predictions: Phase spaces

To more easily visualize the optimal vowel systems that are found for various choices of $\langle \lambda, \alpha \rangle$, Schwartz et al. plot the optimal vowel systems in the $\lambda \times \alpha$ space by means of a "phase space" diagram, which divides the $\lambda \times \alpha$ space into regions where particular vowel systems are found to be optimal:

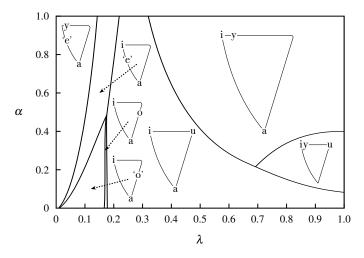
²Random sampling in this search algorithm was done using the runif() and sample() functions in the R programming language (Ihaka and Gentleman 1996).

(10) Schwartz et al.'s phase space for N = 3



Sanders and Padgett's (2008) search algorithm explores the $\lambda \times \alpha$ space more thoroughly, providing a more complete phase space, for all N:

(11) Sanders and Padgett's phase space for N = 3



3 Articulation

3.1 Why articulation matters

Argument 1: The presence versus absence of a contrast affects "markedness". For vowel color, what counts as "unmarked" depends on how many contrastive vowel colors there are (Flemming 1995 [2002]):

3 vowel colors 2 vowel colors 1 vowel color i i u i u i

Comparing three versus two colors, we might conclude that central vowels are more marked than front unround and back round vowels; i.e., $*i \gg *i$, *u.

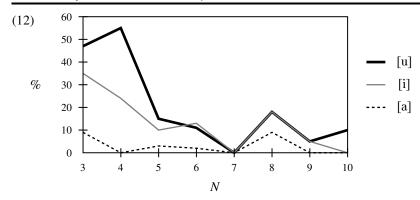
However, if central vowels are truly the least marked vowel color, then it is odd that they show up precisely when a vowel system only has one vowel color, as in so-called "vertical" vowel systems like Kabardian (Choi 1989, 1991) and Marshallese (Choi 1995), where it seems *i, $*u \gg *i$.

We find a similar asymmetry with the markedness of [ə], which is relatively marked when many contrasts exist (*ə \gg *i, *u, *a), but in the absence of contrast (e.g., in reduction contexts), [ə] is common (*ə \gg *i, *u, *a).

Argument 2: DFT's "transparency hypothesis". DFT generally does poorly at generating vowel systems with [ə]. For example, [i e ə a o u] is a relatively common type of 6-vowel system that DFT can't predict as optimal.

Hence Schwartz et al. (1997b) resort to a "transparency" rule for $[\mathfrak{d}]$, essentially stipulating that any DFT-generable system plus $[\mathfrak{d}]$ is a good system. Factoring articulatory ease into DFT's equations might render $[\mathfrak{d}]$ directly generable within DFT.

Argument 3: There seems to be a relationship between number of vowels and extremity of articulation. For example, our preliminary statistics on the absence of the "corner" vowels [i], [a], [u] in relationship to system size show a general downward trend (as well as an interesting asymmetry among the three vowels, with [u] missing more frequently and [a] missing less frequently):



Conclusion: Languages avoid articulatory extremes when they are not necessary, and this should be encoded directly into DFT as a third energy component.

3.2 Adding articulation to DFT

We propose a simple modification to the basic DFT energy equation, adding in a term for articulatory energy E_A :

(13)
$$E_{DFA} = E_D + E_F + E_A$$

where E_A is given by the sum of the individual articulator energies of each vowel in the system (L for lips, H for tongue height, and B for tongue backness):

(14)
$$E_A = \gamma \sum_{i=1}^{N} (L_i + H_i + B_i)$$

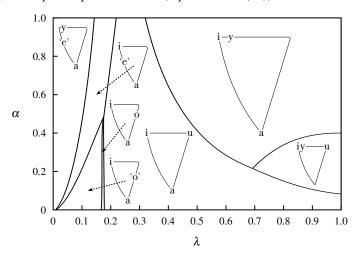
lower $E_A \Leftrightarrow$ more mid/central/neutral vowels

After some initial testing, we started full simulations with a setting of $\gamma = 0.1$ and with the individual vowels articulator energies in (15):

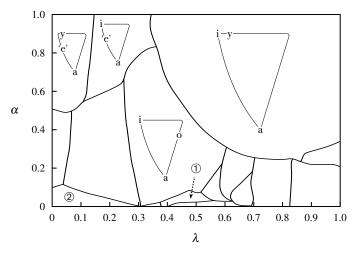
So, the E_A for [i] would be $\gamma(1/2+1+1/2)=0.2$, while ['ə'] has $E_A=0$.

3.3 Comparison of results

(16) DFT phase space for N = 3 (repeated from (11))



(17) DFT+artic phase space for N = 3



Adding our articulatory parameter preserves four of the seven original systems found to be optimal in DFT; these four are shown in full in (17). Of these four systems, none are attested directly in UPSID, though [i a o] is similar to the attested vowel system [i a 'o'] found in Pirahã. (The other three seem unlikely to represent real vowel systems.)

Note however that DFT originally *did* predict [i a 'o'], but this system, along with [i a u] and [i y u], is lost when our articulator parameter is added:

(18) Missing predictions for N = 3

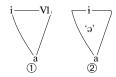


Even though the loss of [i a 'o'] is somewhat troubling, [i a o] is still retained, the basic system type is still predicted to be optimal, so only minor tweaking of the articulatory parameter values seems to be needed.

It could also be the case that precise phonetic measurement of the Pirahã vowel system would reveal that it is actually more similar to the prototype values in [i a o] rather than [i a 'o'].

More problematic is the loss of [i u a], the most common 3-vowel system in UPSID, attested in at least 11 languages, including Tsimshian, Aleut, and Arrernte. The closest system predicted by DFT+artic is the system \oplus [i a V1], one of two new interesting systems in the revised model:

(19) New interesting predictions for N = 3



Many of the attested [i a u] systems may in fact have a more neutral vowel like [V1] than a fully rounded [u], but it seems unlikely that every single one of them has [V1] instead of [u], so some combination of the articulator energies for [u] needs to be adjusted downward to make [u]'s overall articulatory energy small enough for [i a u] to emerge in the predictions.

Note also that $\mathfrak{D}[i \ a \ V1]$ is a step closer to $[i \ a \ \varpi]$, the vowel system of Jaqaru, which DFT does not originally predict.

Finally, the newly predicted vertical system $\mathfrak{D}[i \ 'a' \ a]$ is promising. Nothing like it is predicted by DFT originally, and though this is not attested in UPSID, it is a vertical system of the kind mentioned in §3.1, attested in Kabardian and Marshallese.

Overall, adding an articulatory parameter to DFT yields mixed results for N=3, losing exact matches for two attested system types (one of which is robustly attested), but getting closer to a match for Jaqaru and gaining the ability to predict vertical vowel system.

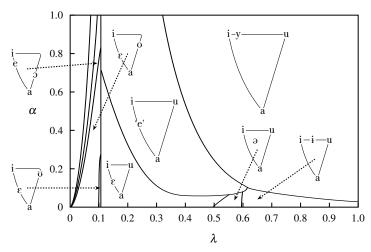
There are still other 3-vowel systems attested in UPSID that are not predicted exactly by DFT with or without articulation. Some of these, like [i a σ] (Haida), [I a σ] (Bella Coola, Caddo, Garawa, and Yanyuwa), and [i ϖ u] (Shilha) are very similar to [i a u], but others are a bit more exotic, and may require significant modifications to be predicted in DFT (20):

(20) UPSID systems not predicted by DFT or DFT+artic

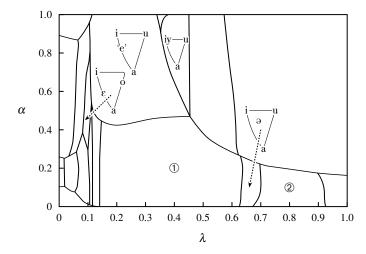
[e a o] Alabama and Amuesha

['ə' a 'o'] Qawasqar

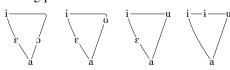
(21) DFT phase space for N = 4



(22) DFT+artic phase space for N = 4



(23) Missing predictions for N = 4

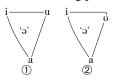


As with N=3, we get mixed results for N=4. Again, there are some systems originally predicted by DFT that are lost with the addition of our articulatory parameter. Two of these ([i ε a υ] and [i ε a υ]) are directly attested in UPSID (Marinhpatha and Moxo, respectively), while the other two or not.

All is not lost however, since we maintain the system [i 'e' a u], which is similar to the two missing predictions attested in UPSID.

Furthermore, we gain at least two new interesting predictions, which are better matches for attested systems in UPSID than DFT's original predictions:

(24) New interesting predictions for N = 4



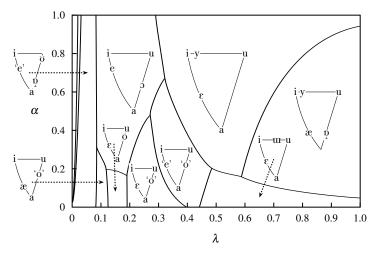
 $\mathbb{O}[i \text{ '$\partial'} \text{ a u}]$ is directly attested in Ivatan, Paiwan, and Yupik, while $\mathbb{O}[i \text{ '$\partial'} \text{ a v}]$ is very similar to the attested system of Lushootseed, $[\text{I '$\partial'} \text{ a v}]$.

There are still other 4-vowel systems attested in UPSID that are not predicted exactly by DFT with or without articulation. Some of these, like [i ϵ α σ] (Cayapa), [i 'e' a 'o'] (Klamath and Takana), and [i 'e' a σ] (Malagasy) are very similar to predicted systems, but others are a bit more exotic, and may require significant modifications to be predicted in DFT (25):

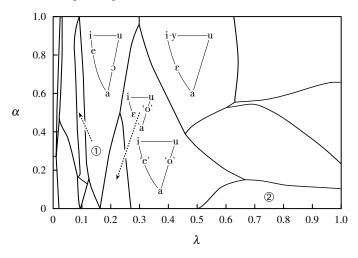
(25) UPSID systems not predicted by DFT or DFT+artic

[i a ɔ u] Tiwi
[I ɐ a ɰ̄] Nunggubuyu
[i ɛ a 'o'] Margi
[i ɜ a ɒ] Yessan-Mayo
['e' 'ə' a 'o'] Upper Chehalis

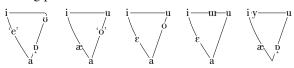
(26) DFT phase space for N = 5



(27) DFT+artic phase space for N = 5



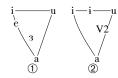
(28) Missing predictions for N = 5



Of the five missing predictions, only one is actually attested in UPSID ([i ϵ a o u], in Jacaltec and Nasioi), but we retain very similar systems, such as [i ϵ a 'o' u]. The other four are not attested directly in UPSID.

Continuing the pattern we've already seen, DFT+artic does make new interesting predictions:

(29) New interesting predictions for N = 5



While these are not attested directly in UPSID, there are similar to attested systems that ordinary DFT cannot approximate: ①[i e 3 a u] is similar to Koya's [i e v a u], while ②[i i a V2 u] is similar to Papago's [i i a 2 u].

There are still other 5-vowel systems attested in UPSID that are not predicted exactly by DFT with or without articulation (30):

(30) UPSID systems not predicted by DFT or DFT+artic

[i i 'e' a 'o'] Abipon [i i ɛ a 'o'] Cofan [i ɛ a ɔ o] Tseshaht [i ø æ a ʊ] Hopi [i ø ɛ a ʊ] Malakmalak ['e' æ ɔ u w] Hixkaryana

4 Wrap-up and future work

The specific method we have used to make DFT sensitive to articulation produces promising, but mixed, results.

Counting hits between predictions and attested vowel systems, we find a moderate increase in the number of languages predicted by DFT+artic. In (31), "hits" are exact matches between prediction and attestation; "near hits" match all but one vowel, which is off by one position in the acoustic space; and "near-ish" hits match all but two vowels, which are each off by one position. Numbers in each cell are the number of hits for systems and languages, respectively.

(31)	N = 3	hits	near hits	near-ish	total
	DFT	2, 12	1, 1	1, 4	4, 17
	DFT+artic	$1, 2^3$	4, 14	1, 4	6, 20
		ı			
	N = 4				
	DFT	4, 5	6, 9	1, 2	11, 16
	DFT+artic	3, 6	4, 5	3, 4	10, 15
	N = 5				
	DFT	3, 51	6, 17	7, 9	16, 77
	DFT+artic	2, 49	5, 17	10, 12	17, 78
		ı			
	N = 6				
	DFT	1, 1	2, 9	6, 9	9, 19
	DFT+artic	1, 1	2, 9	2, 3	5, 13
	N = 7				
	DFT	_	1, 1	4, 4	5, 5
	DFT+artic	_		7, 7	7, 7
	totals				
	DFT	10, 69	16, 37	19, 28	45, 134
	DFT+artic	7, 58	15, 45	23, 30	45, 133

On balance, it seems that the addition of our articulatory parameter doesn't have any major quantitative effect. There is a shift away from exact hits to near(ish) hits, but generally speaking, we still get about the same number of total hits.

Qualitatively, however, there is a noticeable difference. Most of DFT+artic's lost systems are similar to systems it does predict, but it also predicts new systems that belong to entirely different classes than what could be achieved in ordinary DFT, e.g., vertical 3-vowel systems and 5-vowel systems containing central yowels.

To do list:

- Explore different settings for γ (quick pilot experiments suggest that $\gamma = 0.2$ is too high).
- Explore different settings for *L*, *H*, and *B*; for example, varying *B* and/or *L* based on vowel height.
- Explore different kinds of formulas. Rather than summing up the total energy of a system, it might be better to take an average, or look only at minima/maxima in dispersion, focalization, and/or articulation.
- Explore better metrics of "nearness" for counting hits.

Finally, there are deeper questions we want answers for:

- How much can λ, α, and γ really vary across languages? Is the variation independent of *N*?
- Do DFT predictions tell us anything about the relative frequency of vowel systems, the stability of vowel systems, or the likely direction of sound change within a vowel system? Perhaps the comparative size of a phase space region has significance.
- Do near-optimal systems at the same point in a phase space have any interesting predictive status?
- What can we do about "crazy" systems, like Qawasqar's ['e' a 'o']? (Note Wikipedia claims Qawasqar's vowel system is rather ordinary.) Should we even care about trying to get our model to predict them?

³Includes Kabardian and Marshallese, which are not in UPSID.

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